

Zero Temperature Dynamics of the Weakly-Disordered Ising Model

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ABSTRACT

The Glauber dynamics of the pure and weakly-disordered random-bond 2d Ising model is studied at zero-temperature. A single characteristic length scale, $L(t)$, is extracted from the equal time correlation function. In the pure case, the persistence probability, $P(t)$, decreases algebraically with the coarsening length scale.

In the disordered case, three distinct regimes are identified: a short time regime where the behaviour is pure-like; an intermediate regime where the persistence probability decays non-algebraically with time; and a long time regime where the domains freeze and there is a cessation of growth. In the intermediate regime, we find that $P(t) \sim L(t)^{-\theta'}$, where $\theta' = 0.420 \pm 0.009$. The value of θ' is consistent with that found for the pure 2d Ising model at zero-temperature.

Our results in the intermediate regime are consistent with a logarithmic decay of the persistence probability with time, $P(t) \sim (\ln t)^{-\theta_d}$, where $\theta_d = 0.63 \pm 0.01$.

The ‘persistence’ problem is concerned with the determination of the fraction of space which persists in the same phase up to some later time. So, for spin systems we are interested in the fraction of spins that have not flipped in some time t . This problem has been studied extensively over the last few years [1-12] and, somewhat surprisingly, the persistence exponent (θ) has been found to be highly non-trivial even for simple one-dimensional models, such as the q -state Potts model at zero temperature [6-7], of non-equilibrium coarsening dynamics.

Although Stauffer [3] has performed Monte Carlo simulations in up to 5d, most of the work in higher dimensions has been largely limited to 2d. Numerical studies [1,3] estimate that $\theta \sim 0.22$ for the 2d Ising model with Glauber dynamics at $T = 0$. The analogous exponent for non-equilibrium critical dynamics has also come under intensive investigation [9-12]. Very recently, the persistence problem has been generalised to partial survivors [8]. There has, however, been relatively little published in the literature to date on the persistence problem in systems containing disorder. Here we present the results of a numerical study of an Ising model containing quenched impurities.

In this work we study domain growth [13] in a weakly disordered random-bond 2d Ising model and restrict ourselves to zero temperature.

The model we work with is given by

$$H = - \sum_{\langle ij \rangle} J_{ij} S_i S_j \quad (1)$$

where the Ising spins (S_i) are assumed to be on every site of a square $N = 500 \times 500$ lattice with periodic boundary conditions and the summation runs over all nearest-neighbour pairs. The quenched ferromagnetic interactions are chosen from a binary distribution, namely

$$P(J_{ij}) = (1 - p)\delta(J_{ij}) + p\delta(J_{ij} - 1) \quad (2)$$

where p is the concentration of bonds.

The data presented here were obtained on a suite of Silicon Graphics workstations.

We work at zero temperature and consider a range of bond-concentrations in the vicinity of the pure case, $0.975 \leq p \leq 1.0$. The initial configuration of the spins is chosen at random i.e. $S_i(t = 0) = \pm 1$ with equal probability for all i . We then update the lattice using the following algorithm:

1. for a given spin S_i we first calculate the local energy, ΔE_i ;
2. if $\Delta E_i < 0$, we leave S_i as it is;
3. if $\Delta E_i = 0$, we flip S_i at random (i.e. with a probability 1/2);
4. if $\Delta E_i > 0$, we flip S_i with probability 1.

We repeat steps 1 - 4 throughout the entire lattice during each Monte Carlo step.

The number, $n(t)$, of spins which have never flipped until time t is then counted. In practice, we record $n(t = t_r)$ where $t_r = 2^r, r = 0, 1, \dots, 13$.

The persistence probability is defined by [1]

$$P(t) = [\langle n(t) \rangle] / N \quad (3)$$

where $\langle \dots \rangle$ denotes an average over different initial conditions and $[\dots]$ indicates an average over samples i.e. the bond-disorder; typically, *the number of different initial conditions* \times *the number of samples* = 100.

During the simulations we also record the equal time pair correlation function, $C(r, t)$, which is defined by [13]

$$C(r, t) = \frac{1}{N} \sum_i [\langle S_i(t) S_{i+r}(t) \rangle]. \quad (4)$$

According to the scaling hypothesis

$$C(r, t) = f\left(\frac{r}{L(t)}\right) \quad (5)$$

where $f(x)$ is a scaling function and $L(t)$ is a single characteristic coarsening length scale. For the pure model ($p = 1.0$) it's now well established that $P(t)$ decays algebraically [1]

$$P(t) \sim t^{-\theta} \quad (6)$$

where $\theta \sim 0.22$. Furthermore, for the non-random model it's also well known that the domain length increases as $t^{1/2}$ [14].

Hence, from equation (6) we can write

$$P(t) \sim (t^{1/2})^{-\theta'} \quad (7)$$

where $\theta' = 2\theta$.

We now turn to our numerical results. To begin with, we look at the pure ($p = 1.0$) case to extract the value of θ' mentioned above for our model.

In Figure 1 we plot $\ln P(t)$ versus $\ln t^{1/2}$ for the pure case over the time interval $2 \leq t \leq 4096$. The slope of the straight line gives $\theta' = 0.418 \pm 0.004$, which, of course, implies that $\theta = 0.209 \pm 0.002$, consistent with previous results [1,3] (the error-bar quoted here is a statistical one).

As an independent check, we also extracted the coarsening length scale by fitting the equal time correlation function, equation (4), to its expected form, equation(5). Our results are completely consistent with

$$L(t) \sim t^{1/2}. \quad (8)$$

When quenched impurities are introduced the domains grow more slowly than in pure systems and for $T > 0$ it's expected that [15] they increase as $(T \ln t)^x$, where the exponent $x = 4$ in $d = 2$ [16]. For $T = 0$ we expect the quenched disorder to lead to an *eventual* cessation of growth.

As we are working with weakly-disordered models, one would expect the initial decay of $P(t)$ to be given by equation (6). In Figure 2(a) we show a log-log plot of $P(t)$ against t for a range of bond concentrations, $p : 0.975 \leq p \leq 1.0$. Although the initial decrease in $P(t)$ is indeed algebraic, there appears to be non-algebraic decay before ‘freezing’ sets in. This is shown explicitly in Figure 2(b) where deviations from algebraic behaviour can be clearly seen. To investigate this point further, we re-plot the data in Figure 3 as $\ln P(t)$ against $\ln(\ln t)$. As a consequence, we see that the persistence probability decays as

$$P(t) \sim (\ln t)^{-\theta_d} \quad (9)$$

for a disordered system, where θ_d is now the persistence exponent, before the long-time behaviour sets in. Furthermore, we notice that the behaviour for the various disordered cases is qualitatively the same, irrespective of the amount of disorder present.

For $p < 1.0$ three distinct regimes can be identified: an initial short time regime ($t < t_1$) over which the behaviour is pure-like, an intermediate regime ($t_1 \leq t \leq t_2$) over which the persistence probability decreases logarithmically and a final regime ($t > t_2$) where the system appears to ‘freeze’ and $P(t)$ effectively remains constant. It’s clear from Figure 3 that as disorder increases, t_2 decreases, i.e. the cessation of domain growth is quickened by the strength of the disorder. The three different regimes are clearly evident even in a very weakly disordered ($p = 0.99$) system. To ensure that we have a reasonably large intermediate regime to work with, we now restrict our attention to the case where $p = 0.99$.

In Figure 4 we re-plot the data for $p = 0.99$ over the range $16 \leq t \leq 1024$. The data for short times ($t < 16$) has been discarded as has the data over times ($t > 1024$) where the freezing of domains has occurred. The straight line fit leads to a persistence exponent of $\theta_d = 0.63 \pm 0.01$. This result would appear to indicate a logarithmic growth of domains during the intermediate regime at zero-temperature. This is somewhat surprising as the logarithmic behaviour discussed earlier is believed to hold true for *finite* temperatures.

The behaviour of the growth before ‘freezing’ sets in can be extracted

independently by fitting the equal time correlation function to its expected scaling form given by equation (5). In Figure 5 we present the scaling plot of $C(r, t)$ for $p = 0.99$. We plot $C(r, t)$ against $r/L(t)$, where $L(t)$ has been chosen at each time to give the best data collapse. This clearly produces an excellent scaling plot. We stress that the plot shown in Figure 5 makes no assumptions about the growth law of $L(t)$ with t . In Figure 6 we plot the data for the persistence probability for $p = 0.99$ as a log-log plot of $P(t)$ against $L(t)$ where the latter has been extracted from Figure 5. The linear fit in Figure 6 implies that $\theta' = 0.420 \pm 0.009$, consistent with our earlier result for the *pure* case. Thus, expressing $P(t)$ in terms of the coarsening length scale leads to the same behaviour as for the pure case.

To conclude, we have presented data for the zero-temperature dynamics of the weakly disordered random-bond 2d Ising model. For the disordered system we find evidence that $P(t)$ decreases logarithmically with time over an intermediate regime. The (disordered) persistence exponent over this regime is estimated to be $\theta_d = 0.63 \pm 0.01$. However, for both the pure and the disordered models the persistence probability is found to decay algebraically with the coarsening length scale with the *same* exponent. At present we are studying generalised persistence [8-12] for disordered models.

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FIGURE CAPTIONS

Figure 1

A log-log plot of $P(t)$ against $t^{1/2}$, for the pure 2d Ising model. The linear fit shown gives a value of $\theta' = 0.418 \pm 0.004$.

Figure 2(a)

A plot of $\ln P(t)$ against $\ln t$ for a range of bond-concentrations, p ; the data for the pure case, $p = 1.0$, is plotted for comparison and the straight line has slope -0.209 .

Figure 2(b)

A re-plot of some of the data shown in Figure 2(a) on an expanded scale to highlight the deviations from algebraic decay; the linear fit is for the pure case with $\theta = 0.209$.

Figure 3

A plot of $\ln P(t)$ against $\ln(\ln t)$ for a range of bond-concentrations, p ; the data for the pure case, $p = 1.0$, is plotted for comparison.

Figure 4

A re-plot of the data for $p = 0.99$ from Figure 3. The straight line fit confirm a logarithmic decay of the persistence probability over time. The gradient of the line shown leads to $\theta_d = 0.63 \pm 0.01$.

Figure 5

A scaling plot of $C(r, t)$ versus $r/L(t)$, where $L(t)$ has been chosen at each time to give the best data collapse.

Figure 6

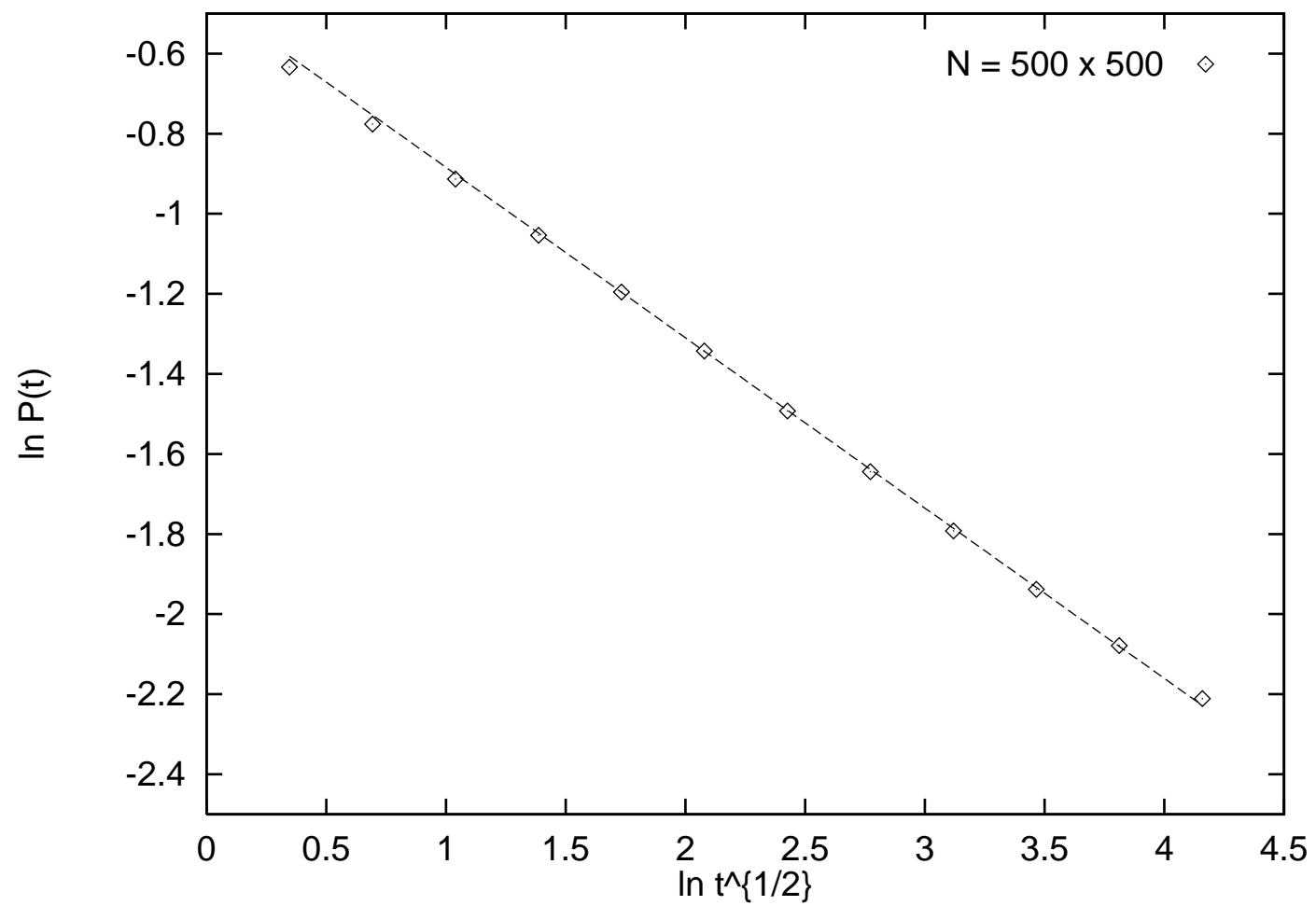
A plot of $\ln P(t)$ versus $\ln L(t)$, where $L(t)$ has been extracted from Figure 5. The linear fit confirms that the persistence probability decays algebraically with the coarsening length scale, $L(t)$. The slope of the straight line yields $\theta' = 0.420 \pm 0.009$.

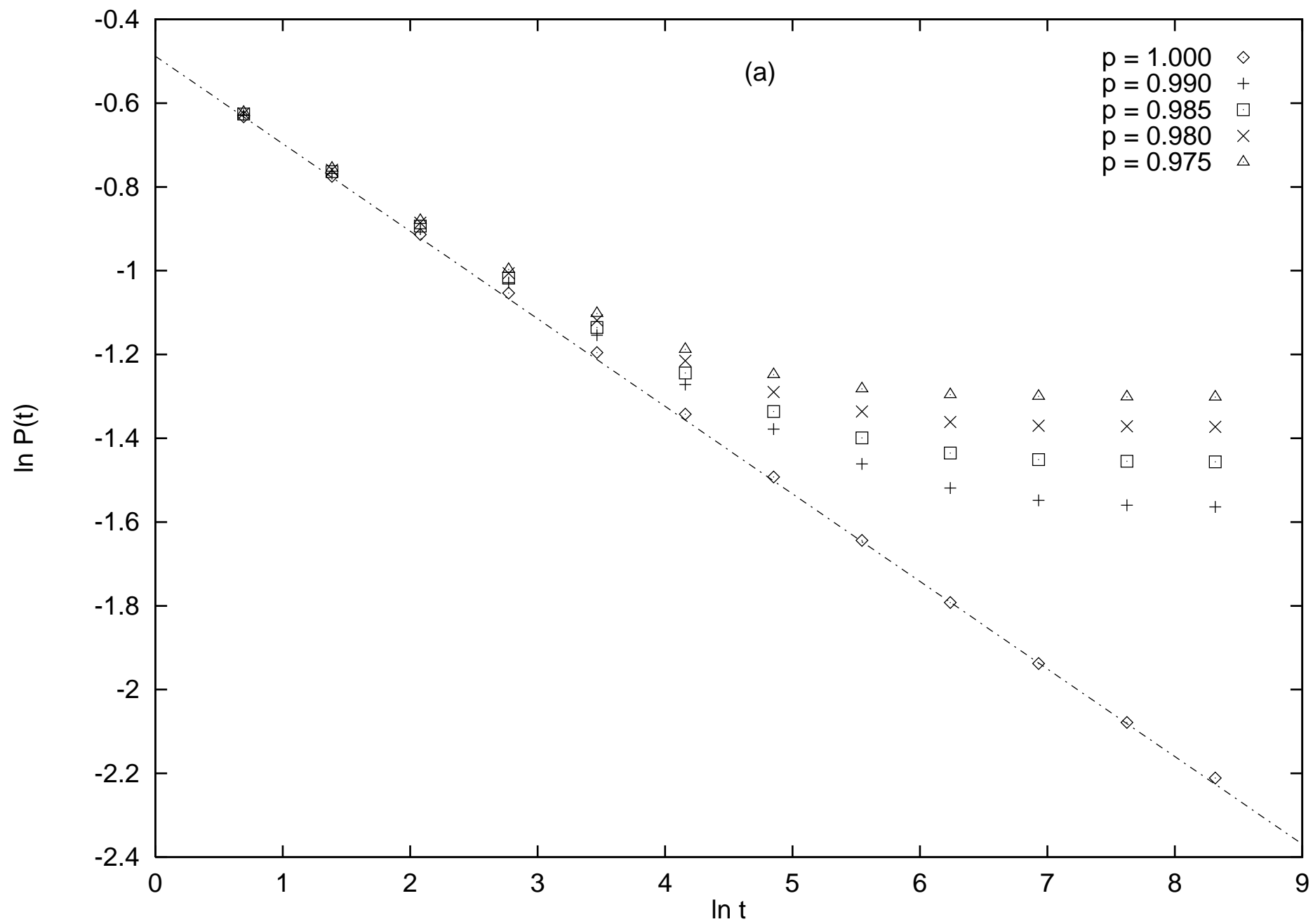
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Figure 1





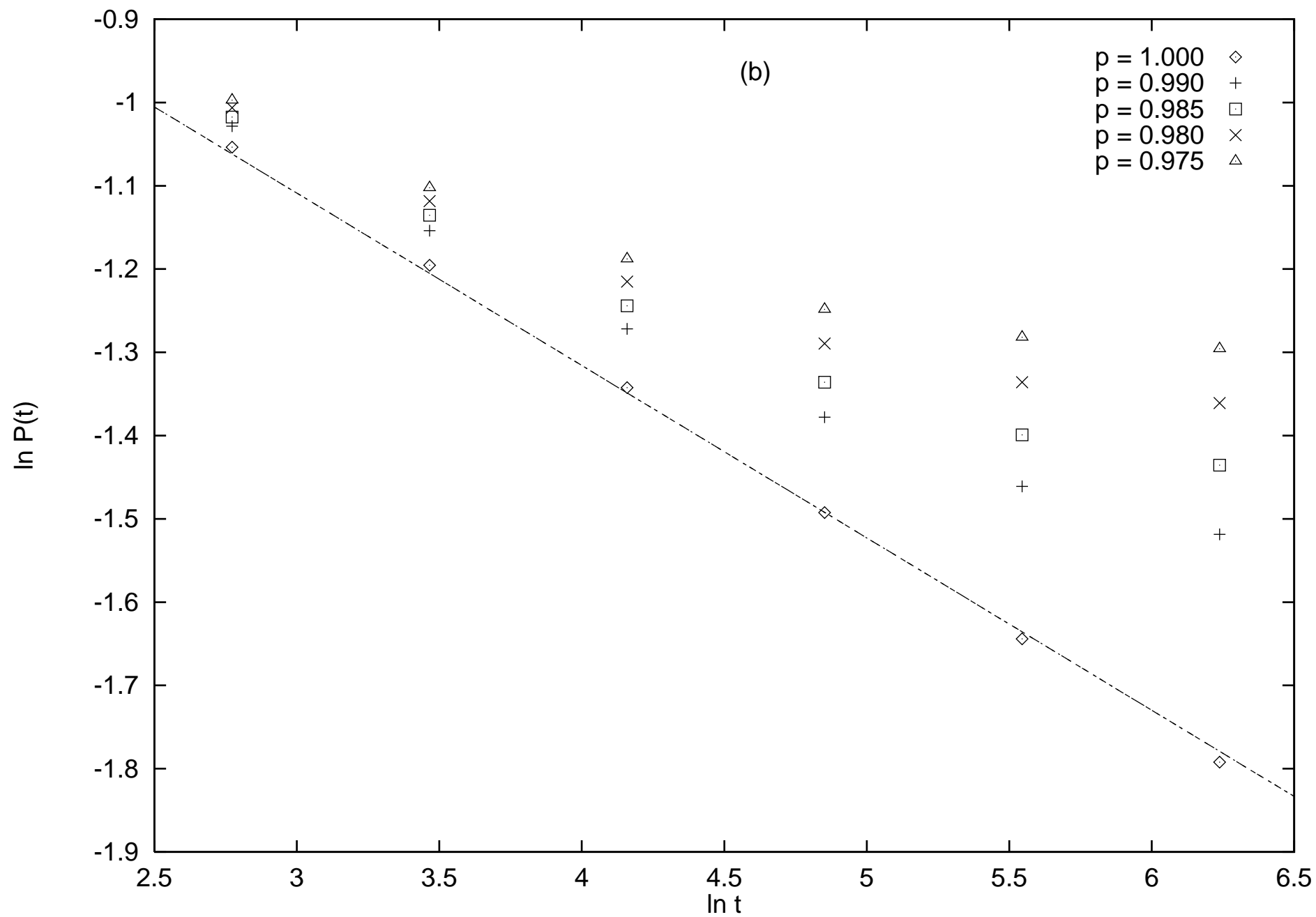


Figure 3

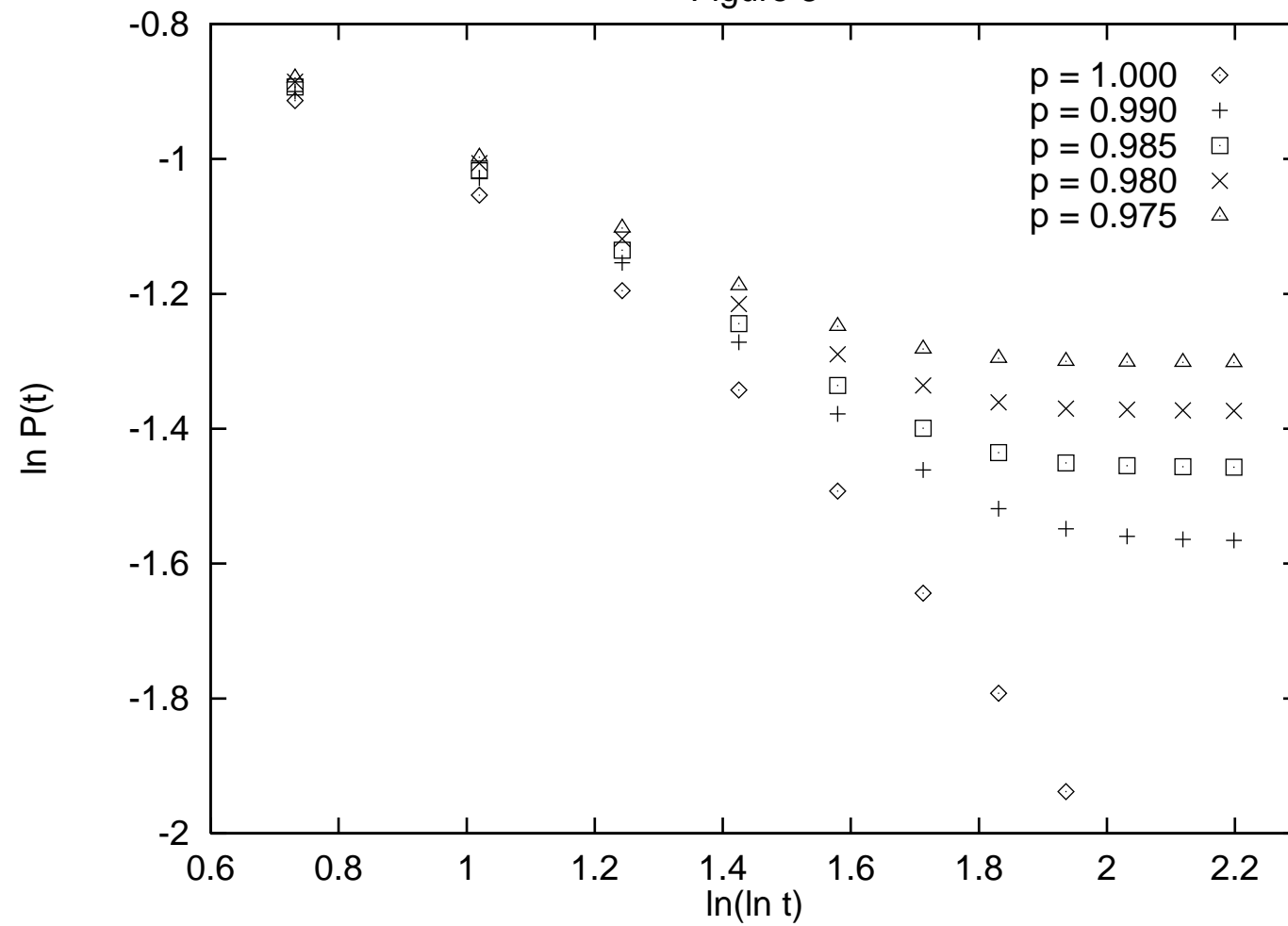


Figure 4

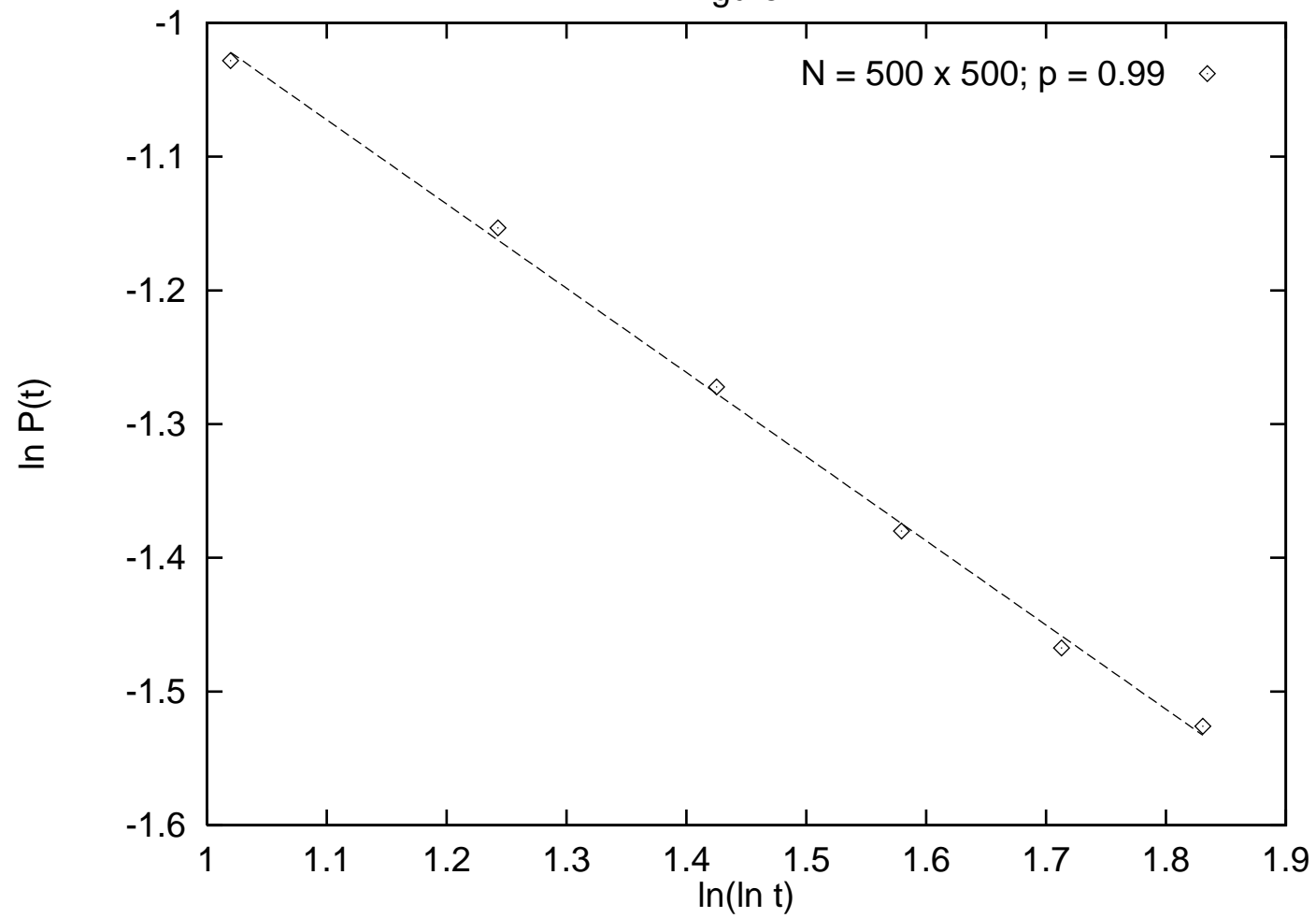


Figure 5

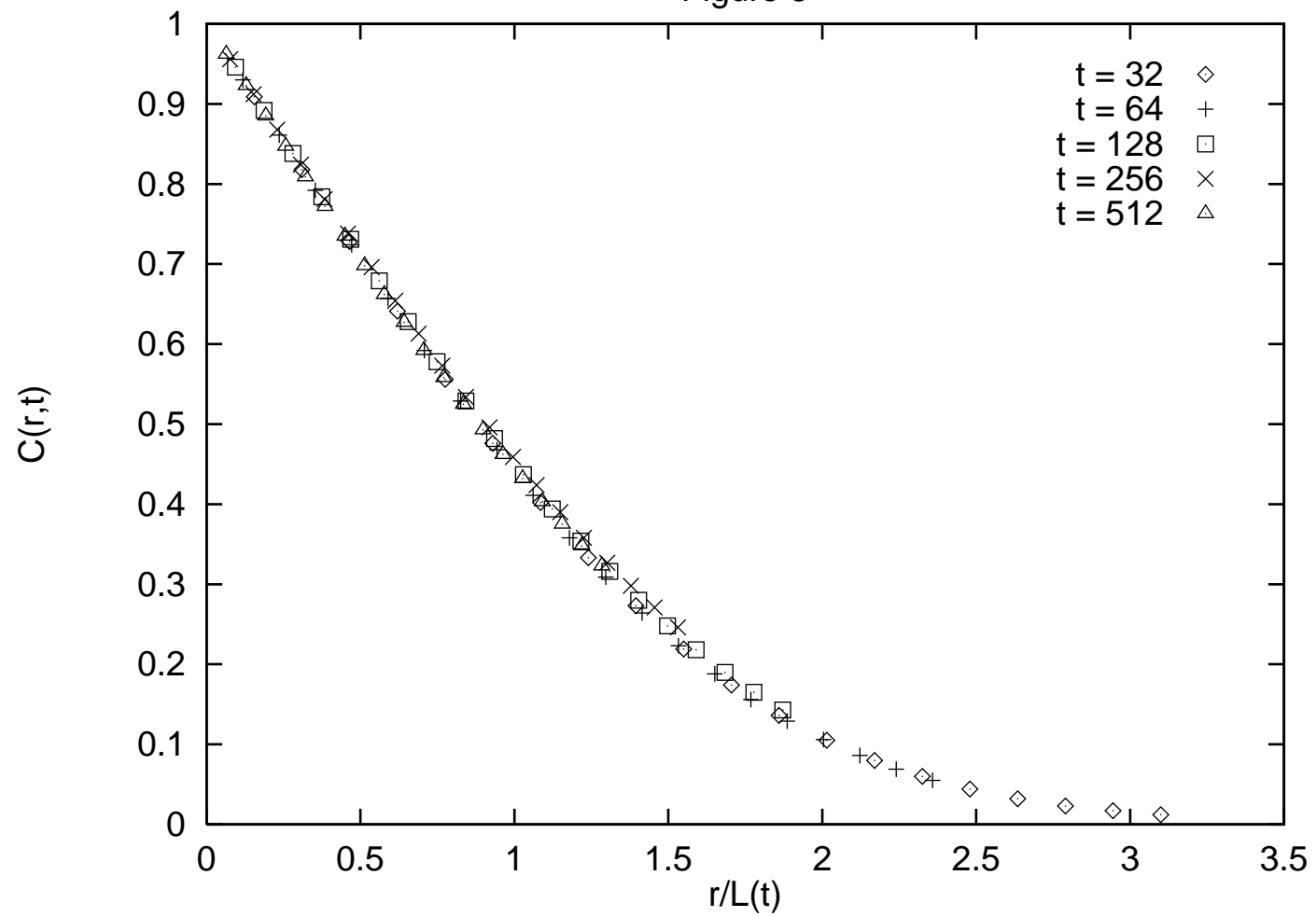


Figure 6

